A Deterministic Quasi-Static Approach to Microstrip Discontinuity Problems in the Space-Spectral Domain

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***Abstract*—A new deterministic quasi-static space-spectral domain approach has been developed to analyze planar circuit discontinuities. This method represents an important modification of the SSDA which combines the advantages of the 1-D MoL and the 1-D SDA. Since the SSDA was developed as an eigenvalue approach, it was only capable of calculating resonant frequencies of planar resonators of arbitrary shape. In this modified SSDA approach an algebraic matrix equation replaces the eigenvalue matrix and the quasi-static discontinuity capacitances and *s*-parameters are computed very efficiently. To demonstrate the performance of this new method the microstrip open-end is calculated as an example.**

I. Introduction

Accurate characterization of planar circuit discontinuities is important in the design of MMIC's and Miniature Hybrid MIC's (MHMIC's). In general, a rigorous discontinuity analysis requires complicated and CPU-time consuming numerical techniques. In many cases, however, a quasi-static approach to determine equivalent circuit parameters can be an accurate and efficient alternative, at least for transmission line structures with low dispersion (i.e., microstrip and coplanar waveguide) at frequencies below 25 GHz [1]. In this letter, we introduce a deterministic quasi-static approach in the space-spectral domain to solve planar circuit discontinuity problems. This new approach has several advantages over other numerical techniques. Besides being numerically very efficient, it can treat arbitrarily shaped planar discontinuities.

The method is based on the space-spectral domain approach (SSDA) which was first introduced by Wu and Vahldieck [2] for the calculation of resonant frequencies of arbitrarily shaped microstrip resonators. The SSDA algorithm leads to an eigenvalue problem in which the zeros of the determinant of the system matrix provided the resonant frequency of the planar structure. This approach is sometimes numerically unstable and does not lead directly to the *s*-parameters of discontinuites.

In this letter, the SSDA algorithm is modified to calculate directly the quasi-static capacitances and *s*-parameters of a class of microstrip discontinuities by using a new deterministic technique. To minimize errors in the calculation of the capacitance parameters, the excess charge density [5] has been used and calculated in the space-spectral domain in one step via Galerkin's method. This approach leads to an algebraic equation for the *s*-parameters rather than to an eigenvalue equation as in the original SSDA. The new quasi-static SSDA algortihm is numerically stable, requires little memory space and is very fast on serial computers. Although this method is capable of treating arbitrarily shaped discontinuities, we are concentrating in this letter only on the simple discontinuity shown in Fig. 1, to demonstrate this technique. Therefore, as an example, equivalent capacitances of the microstrip open end are analyzed. The agreement with published results is very good. To include arbitrarily changing circuit contours in *z*-direction requires a changing set of basis functions per line in *z*-direction.

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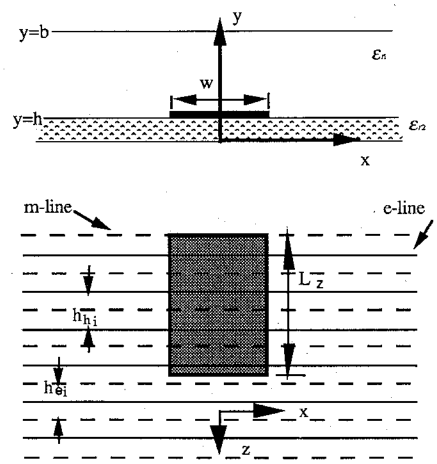

Fig. 1. Cross-section of the microstrip open end and discretization in *z*-direction.

II. Theory

In general, quasi-static modeling of planar discontinuity problems is based on the 3-D Laplace's equation $\nabla^2\Phi = 0$, which contains the three spatial variables. To eliminate two of them the sequence of steps in the quasi-static SSDA is as follows (see Fig. 1). First transform the electric potential function Φ into $\tilde{\Phi}$ via a Fourier transform along the *x*-direction, then discretize the *z*-direction that leads to the vector $\overline{\tilde{\Phi}}$ and finally

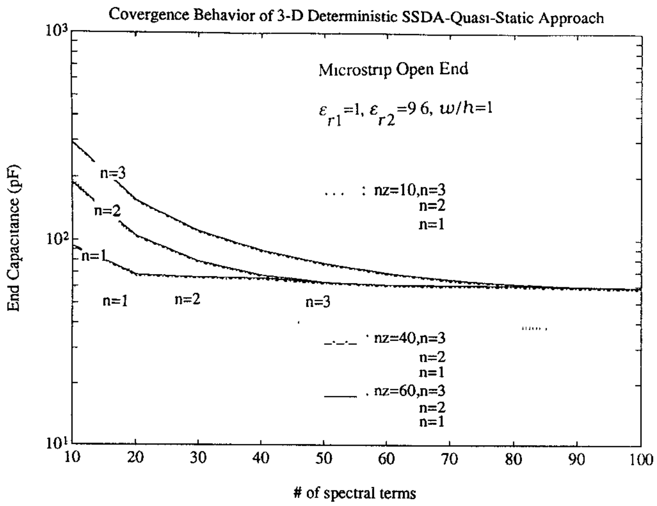

Fig. 2. Convergence behavior.

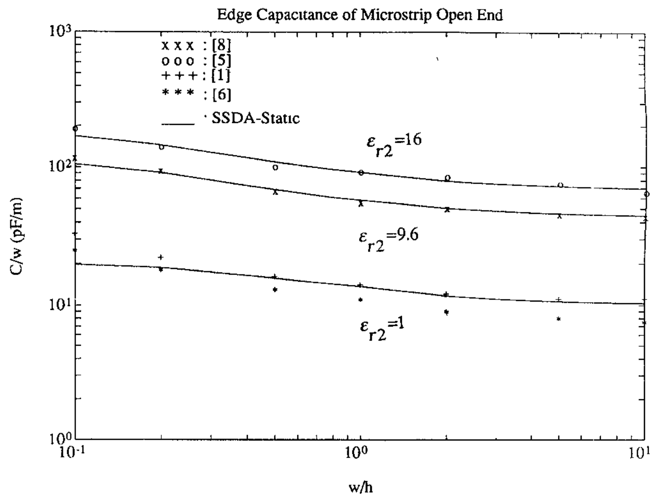

Fig. 3. Microstrip open end capacitances versus microstrip dimensions.

transform $\overline{\Phi}$ into $\overline{\psi}$ by using an orthogonal transformation to decouple Laplace's equation. A microstrip open end illustrated in Fig. 1 is used as an example to demonstrate the theory. The transformation can be expressed as

$$\int \Phi e^{jax}\, dx = \tilde{\Phi}, \qquad \frac{\partial^2}{\partial x^2}\Phi \rightarrow -\alpha^2\tilde{\Phi} \tag{1}$$

after the Fourier transform and

$$\overline{\Phi} = \left[\tilde{\Phi}_1 \cdots \tilde{\Phi}_{nz}\right]^t, \qquad h_0^2\frac{\partial^2}{\partial z^2}\overline{\Phi} \rightarrow -[D_{zz}]\overline{\psi} \tag{2}$$

after discretization and orthogonal transform, where

$$[D_{zz}] = [D_z]^t[D_z],$$
$$[D_z] = \left[r^h\right][D][r^e], \qquad [D] = \begin{bmatrix} -1 & 1 & & \\ & \ddots & \ddots & \\ & & -1 & 1 \end{bmatrix}; \tag{3}$$

$$[r^e] = \mathrm{diag}\left\{\sqrt{\frac{h_0}{h_{e_i}}}\right\},$$
$$\left[r^h\right] = \mathrm{diag}\left\{\sqrt{\frac{h_0}{h_{e_i}}}\right\}, \qquad [T]^t[D_{zz}][T] = \delta^2; \tag{4}$$

nz represents the number of discrete lines used in z-direction, $h_0 = Lz/nz$. The 3-D Laplaces equation is simplified to the following differential equation:

$$\frac{\partial^2\overline{\psi}}{\partial y^2} - \gamma^2\overline{\psi} = 0, \qquad \gamma^2 = \alpha^2 + \delta^2. \tag{5}$$

Solutions to (5) can be expressed in terms of the sum of hyperbolic functions. In order to avoid the errors arising from the subtraction of two electric charges which are close in magnitude, the excess charge density [1] is used for solving the 2-D transmission line problem in the spectral domain. This corresponds to (5) with $\delta = 0$. By imposing the boundary condition at the interface of two layers ($y = h$) and removing those charge densities related to an infinite transmission line, an equation governing the relationship between the potential and the excess charge density can be derived. Upon the orthogonal transformation back into the original domain, the equation is written as

$$\overline{\Phi} = \overline{G}(\alpha)\overline{\sigma}, \tag{6}$$

where

$$\overline{G}(\alpha) = \left[r^h\right]^{-1}[T]^t(g(\gamma) - (\alpha))^{-1}[T]\left[r^h\right]$$
$$g(\beta) = \mathrm{diag}\left\{\frac{\varepsilon_{r1}\beta}{\tanh(\beta(b-h))} - \frac{\varepsilon_{r2}\beta}{\tanh(\beta h)}\right\}$$
$$\beta = \gamma \text{ or } \alpha. \tag{7}$$

The excess charge density is expanded as the sum of n basis functions

$$\sigma_1(x) = \frac{\cos\left[2(i-1)\pi\frac{x}{w}\right]}{\sqrt{1-\left(\frac{2x}{w}\right)^2}}, \qquad i = 1, 2, \cdots, n. \tag{8}$$

With Parsevals theorem in mind

$$\int \Phi(\alpha)\sigma_i(\alpha)\, d\alpha = 2\pi V\sigma_i(\alpha = 0) \tag{9}$$

and after applying Galerkins technique in the spectral domain for each discrete line, the coefficients of the excess charge density can be obtained by one matrix inversion of (6) instead of finding the zeros of an eigenvalue matrix as in the original SSDA procedure. The end capacitance is then calculated from $C = Q/V$ assuming the voltage $V = 1$ on the strip.

III. Results And Discussion

The convergence behavior of this deterministic algorithm is shown in Fig. 2. A fast and stable convergence characteristic is observed in terms of the number of lines nz, number of basis functions n and number of spectral terms. As the figure shows, there is no difference in convergence between $nz = 40$ and $nz = 60$. The relative convergency behavior [2], [10] of SDA can be also observed in terms of number of basis functions and spectrum terms. The microstrip open end capacitances for different substrates are calculated and compared with results from the literature [1], [5]–[7]. Fig. 3 indicates a

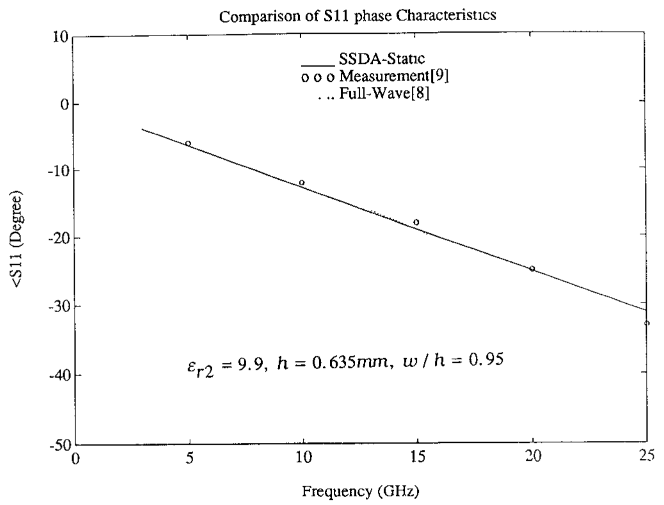

Fig. 4. Phase of S11 versus frequency.

good agreement. Fig. 4 shows the phase characteristic of S_{11} together with measured results [9] and a full-wave analysis [8]. Also here the agreement is excellent. The typical computation speed is 3 seconds per frequency.

IV. Conclusion

This letter has introduced a new deterministic quasi-static approach in the space-spectral domain to calculate microstrip open end capacitances and s-parameters. A comparison with full wave analysis methods and measurements shows excellent agreement. The flexibility and numerical efficiency of this method makes it an attractive CAD tool for frequencies below 25 GHz.

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